Workshop on Evolution Equations, Asymptotic Methods and Generalized Functions

Louisiana State University, March 23-24, 2000

Abstracts

Boris Bäumer (University of Nevada, Reno): *Fractional transport in porous media*

Transport of solutes through porous media is traditionally modeled by Brownian motion with drift. While that might be adequate for the bulk of the plume, the model fails to predict fast and very slow traveling solutes that are reported in numerous field studies. We develop a parametrically simple model using fractional powers of linear operators. It is based on the central limit theorem and predicts heavy tailing, a power law dependence of test scale and conductivity, and super-Fickian growth of the plume. We apply the model to data of the MADE site at Columbus AFB in Mississippi.

Ralph deLaubenfels (Scientia Research Institute, Athens): *I. Asymptotic stability and chaos*

I will present a pointwise Hille-Yosida characterization of stable strongly continuous semigroups. Also, for a decomposable operator A generating a strongly continuous semigroup e^{tA}, e^{tA} has exponentially decaying orbits for all initial data in a dense set if and only if the local spectral subspace for A^*, $X^*(A^*, Re(z) \geq 0)$, is trivial. (These results are from joint work with Vũ Quôc Phóng and Shengwang Wang.)

By way of contrast, for B equal to the left-shift on weighted ℓ^p spaces or differentiation on weighted L^p spaces, those f for which $f(B)$ is chaotic or generates a chaotic semigroup are characterized. (This is joint work with Hassan Emamirad.)

I will discuss the following open question. For what families of operators $\mathcal{A}$ is it the case that, for any $T \in \mathcal{A}$, T is chaotic if and only if T has a nontrivial fixed periodic point?

II. Spectral mapping for operators with polynomially bounded resolvent

For operators A with polynomially bounded resolvent outside $\Omega \subseteq \mathbf{C}$, f holomorphic on a neighborhood of Ω, two sufficient conditions for spectral mapping

$$f(\sigma(A)) \subseteq \sigma(f(A)) \subseteq f(\sigma(A)) \cup \{0\}$$

are given. It is sufficient that

1. $\rho(f(A))$ be nonempty and $|z|^s f(z) \to 0$ as $|z| \to \infty$ in Ω, for some $s > 0$, or

2. $f(A)$ be in the Banach algebra generated by the resolvents of A and f be polynomially bounded on Ω.

Applications to strongly continuous semigroups, integrated semigroups and regularized semigroups are given. I will discuss the following open question.

If A has polynomially bounded resolvent outside $\Omega \subseteq \mathbf{C}$, f is holomorphic on Ω, $|f(z)| \to 0$ as $|z| \to \infty$ in Ω and $\rho(f(A))$ is nonempty, does spectral mapping $f(\sigma(A)) \subseteq \sigma(f(A)) \subseteq f(\sigma(A)) \cup \{0\}$ hold? A special case would be spectral mapping for the strongly continuous semigroup e^{tA} generated by A: $\sigma(e^{tA}) \subseteq e^{t\sigma(A)} \cup \{0\}$ when $\sigma(A) \subseteq \{x+iy \,|\, |y| \leq \phi(|x|),\, x \leq \omega\}$ for some real ω, $\phi : [-\omega, \infty) \to \mathbf{R}$ increasing to ∞, with the resolvent of A polynomially bounded outside that set.

J.R. Dorroh (LSU): *The Study of Semigroups of Transformations in a Polish Space Through Induced Semigroups of Linear transformations on Functions and Measures.*

Let X be a separable, complete metric space (= Polish space), and let $\{T(t) : t \geq 0\}$ be a jointly continuous semigroups of transformations in X. One can consider semigroups $\{U(t)\}$, $\{V(t)\}$ of linear transformations defined on the space $C(X)$ of bounded continuous real-valued functions on X and on the space $M(X)$ of real Borel measures on X, respectively, by $U(t)f = f \circ T(t)$ and $V(t)\mu = \mu \circ T(t)^{-1}$. We characterize semigroups U and V that arise in this way, as well as their infinitesimal generators.

Franziska Kühnemund (Tübingen): *Comments on the Lie-Trotter product formula*

We show that in general the semigroup generated by the closure of the sum of two generators can not be represented by the Lie-Trotter product formula. Furthermore, we give commutator conditions implying the convergence of the Lie-Trotter products.

Günter Lumer (University of Mons-Hainaut, Mons, Belgium and Solvay Institutes for Physics and Chemistry, Brussels, Belgium): *(I) Asymptotic methods, hyperfunctions, and generalized Laplace transforms. Applications to physics, pde's, and computation. (II) Asymptotic expansions of solutions of ODEs, PDEs and evolution equations. (III) Asymptotic expansions for local regularized solutions (in the context of Beals/Chazarin type systems).*

We first recall/survey some recent results on mathematical and physical effects, mostly associated to wave packets in systems governed by parabolic equations, which as far as we know can only be shown to happen, be explained and analyzed, by using together with asymptotic methods, hyperfunctions, δ-expansions and generalized Laplace transforms ($L.T.$ hereafter). These effects concern : nondetectable signals and in the opposite direction identification by "physically visible effects" of true (i.e. non distributional) hyperfunctions; blow up of solutions (and residual effects in form of generalized functions); multiple-valuedness and interpretation of such residual effects after blow up, in parabolic systems. Next, we give very recent basic and applied results concerning generalized $L.T.$ and asymptotic expansions :

(1) We show that the recently introduced (1999) theory of asymptotic $L.T.$, L, by G. Lumer and F. Neubrander, leads to the same objects as the Komatsu $L.T.$, $\mathcal{L}$, indeed $L = \mathcal{L}$, with advantages for computability (in particular on L^1_{loc} L is "more classically accessible", and essentially computable and invertible on the real axis);
(2) We give very recent results on asymptotic and exact expansions of solutions (in X Banach) for equations of the type $u' = Au + g(t)$, $u(0) = 0$, where g may be only L^1_{loc},

explicitely when an asymptotic (finite or infinite) expansion of g at $t = 0$ is known. For A a semigroup generator one has $(Lu)(\lambda) \approx R(\lambda, A)(Lg)(\lambda)$, and one can use $R(\lambda, A) \sim 1/\lambda + A/\lambda^2 + \cdots + A^n/\lambda^{n+1} + \cdots$ with sufficient regularity (in terms of $D(A^n)$). We get exact (instead of asymptotic) expansions with sufficient regularity of $g(t)$. Applications are shown for PDEs, and ODEs.
Much of the results on asymptotic expansions is then shown to extend to situations where A generates only integrated semigroups and even only local K-regularized semigroups (of the type studied earlier by Arendt, Lions, Beals, Chazarin, Cioranescu-Lumer,. . ., including distribution, ultradistribution and hyperfunction semigroups). Part of the above is joint work with Frank Neubrander.

Naoki Tanaka (Okayama University, Japan) *Wellposedness of linear, non-autonomous equations in the sense of Hadamard*

This talk is concerned with evolution equations of the form $u'(t) = A(t)u(t) + f(t)$, where $f \in L^p([0, T]; X)$ for some $1 \leq p < \infty$ and $A(t), t \in [0, T]$ is a family of closed linear operators satisfying a new type of stability condition.

Stephen J. Watson (LSU): *On Temporal Asymptotics of the Dirichlet Boundary Value Problem for the p'th power gas law*

We consider the compressible Navier-Stokes equations for a p'th power gas law. We obtain new a priori bounds on the density for the Dirichlet initial-boundary value problem. This work is part of a broader program aimed at understanding the temporal asymptotics for materials which may change phase; e.g. Van-der-Waals fluid.

Yu Zhuang (LSU): *Classical unstable approximation for linear evolution equations and applications*

Temporal discretization methods for evolutionary partial differential equation that factorize the resolvent into a product of easily computable operators have great numerical appeal, in particular for large scale parallel computing. However, as many other factorized approximation methods that exhibit numerical stability, these methods are known to satisfy only the Von Neumann stability condition, a necessary condition that is usually surmised as sufficient in practical cases as pointed out by Lax and Richtmyer. Thus, to better understand the Von Neumann condition, we investigate the relation between stability and convergence in directions not covered by the Lax equivalence theorem or the Courant-Friedrichs-Lewy condition. To do that, we extend the Trotter-Kato theorem and the Chernoff product formula to possibly unstable approximation sequences and indicate how our results can be used for some unstable factorized approximation methods.