Homework 3

(1) Calculate $\pi_2(S^2 \vee S^1)$.

Hint: use covering spaces

(2) Let $p: \tilde{X} \to X$ be a regular covering map, with $X, \tilde{X}$ connected and locally path connected. Let $\tilde{x}, \tilde{y}, \tilde{z}$ be points of $\tilde{X}$, such that $p(\tilde{x}) = p(\tilde{y}) = p(\tilde{z})$ and let $g: \tilde{X} \to \tilde{X}$ be the covering transformation that maps $\tilde{y}$ to $\tilde{z}$. Explain how to construct $g(\tilde{x})$ by lifting suitable paths and prove that your construction is correct.

(3) Let $C = \{C_n, \partial_n\}$ and $D = \{D_n, \partial_n\}$ be chain complexes and let $f : C \to D$ be a chain map. Let $E_n = C_{n-1} \oplus D_n$ and define $\partial : E_n \to E_{n-1}$ by $\partial(x,y) = (\partial x, fx - \partial y)$. Show that $E = \{E_n, \partial_n\}$ is a chain complex, and that if all the homology groups of E are zero, then f induces isomorphism: $f_* : H_n(C) \to H_n(D)$

(4) Prove that $H_n(X \times D^k, X \times \partial D^k) \cong H_{n-k}(X)$ for any space X and all n, k

(5) Let (X, A) be a pair of spaces, $i : A \to X$ - inclusion. Give a proof or counterexample:

a) If $H_n(X, A) = 0$ $0 \le n \le k$ then $i_* : H_n(A) \to H_n(X)$ is an isomorphism for $0 \le n \le k$.

b) If $i_*: H_n(A) \to H_n(X)$ is an isomorphism for $0 \leq n \leq k$ then $H_n(X,A) = 0$ $\forall\ 0 \leq n \leq k$.