Homework 4

(1) Let $X = S^1 \cup_f D^2$, where $f: \partial D^2 \to S^1$ has degree n. Calculate the homology groups of X and its universal cover $\tilde{X}$.

(2) Prove that if a finite CW complex X retracts onto its n-skeleton X^n, then the boundary map $\partial: C_{n+1}^{CW}(X) \to C_n^{CW}(X)$ is zero.
Is the converse true? (Suggestion: consider $X = S^1 \times S^1$)

(3) Calculate all the homology groups $Y_{nk} = S^n \cup_\phi e^{n+1}$ where the attaching map has degree k.

Prove that for any finitely generated Abelian groups there is a space X, such that $H_0(X) \cong \mathbb{Z}$ and $H_n(X) \cong G_n$ for all $n \geq 1$

(4) Calculate the homology groups of the quotients of $I \times I$ by the equivalence relations

a) $(t,0) \sim (t,1) \sim (0,t) \sim (1,t)$ $\forall t \in I$

b) $(t,0) \sim (1-t,1) \sim (0,1-t) \sim (1,t)$ $\forall t \in I$

(5) Let C be a simple closed curve on $\mathbb{RP}^2$ such that C is not null-homotopic in $\mathbb{RP}^2$. Let X be the space obtained from $\mathbb{RP}^2$ and a torus $S^1 \times S^1$ by identifying C with $S^1 \times \{a\}$ where $a \in S^1$. Calculate the homology groups of X.

(6) Let S^k be the k-sphere and let $f: S^k \to S^n$ be an embedding for $k<n$. Compute

$$H_i(S^n - f(S^k)) \quad \forall\, i$$

Hint: use Mayer-Vietoris sequence.