Homework 5

(1) a) Which maps from $H_2(S^1 \times S^1)$ to $H_2(S^2)$ are induced by continuous maps

$$f: S^1 \times S^1 \to S^2$$

b) Which maps from $H_2(S^2)$ to $H_2(S^1 \times S^1)$ are induced by continuous maps

$$f: S^2 \to S^1 \times S^1$$

(2) Calculate $H^n(T^3; \mathbb{Z})$ of 3-torus. For any map $\alpha: T^3 \to T^3$ calculate the induced maps $\alpha^*: H^n(T^3; \mathbb{Z}) \to H^n(T^3; \mathbb{Z})$ for $n > 1$ in terms of matrix for

$$\alpha^*: H^1(T^3; \mathbb{Z}) \to H^1(T^3; \mathbb{Z})$$

3. Calculate the homology groups of the space $\mathbb{R}P^n/\mathbb{R}P^m$, obtained from real projective n-space $\mathbb{R}P^n$ by identifying all points of the subspace $\mathbb{R}P^m$ to a single point.

4. Let G be a group of homeomorphisms of S^n such that for each $g \in G$, either $g=1$ or $g: S^n \to S^n$ has no fixed points. Prove that if n is even, then G has at most two elements. What happens if n is odd?
(Hint: use Lefschetz fixed point theorem)

5. Prove that $\mathbb{C}P^\infty$ is $K(\mathbb{Z},2)$

6) Show that $\pi_k(SO(n-1)) = \pi_k(SO(n))$ for $k < n-2$, where $SO(m)$ is a group of orthogonal $m \times m$ matrices with determinant $= 1$.

Hint: Show that $(SO(n), S^{n-1}; SO(n-1))$ form a bundle (E, B, F). To do that look at the action of $SO(n)$ on S^{n-1}.