Minimal models and Coulomb gas formalism (Lecture III)

When $c>25$ α_+, α_- are imaginary and

$n, m \to \infty$ $h(n,m)$ will become negative

$25 > c > 1$ - dimensions are complex

$c \leq 1$ - dimensions are real

$c=1$, $\alpha_0 = 0$ $\alpha_\pm = \pm 1$, repres. with null vectors are those such that $h = \frac{N^2}{4}$ $N \in \mathbb{Z}$

α_-/α_+ is rat. number $\to$ finite set of operators

$$\frac{\alpha_-}{\alpha_+} = -p/q \qquad c = 1 - \frac{6(p-q)^2}{pq}$$

$$h(n,m) = \frac{1}{4pq}\left[(nq-mp)^2 - (p-q)^2\right]$$

Unitary subseries: $q = p+1$ $p \geq 2$

$$c = 1 - \frac{6}{p(p+1)}, \quad h_{n,m} = \frac{1}{4p(p+1)}\left((n(p+1)-mp)^2 - 1\right)$$

$$h(n,m) = h(p-n, q-m)$$

$$0 < n < p \quad 0 < m < q \quad , p < q$$

$$p=3, q=4 \quad c = 1/2$$

Ising model

$h(1,1) = h(2,3) = 0$

$h(1,2) = h(2,2) = 1/16$

$h(2,1) = h(1,3) = 1/2$

$\phi_{(1,1)} = \phi_{(2,3)} = Id$

$\phi_{(1,2)} = \phi_{(2,2)} = \sigma$ - spin field

$\phi_{(2,1)} = \phi_{(1,3)} = \varepsilon$ - energy dens.

$$\varepsilon \cdot \varepsilon = \phi_{(2,1)} * \phi_{(2,1)} = c_1[\phi_{(1,1)}] + \underset{0}{\underset{\shortparallel}{c_2}}[\phi_{(3,1)}]$$

$$= \phi_{(1,3)} \times \phi_{(1,3)} = c_1'[\phi_{(1,1)}] + \underset{0}{\underset{\shortparallel}{c_2'}}[\phi_{(1,3)}] + \underset{0}{\underset{\shortparallel}{c_3'}}[\phi_{1,5}]$$

$[\varepsilon]\times[\varepsilon] = [1]$ $\quad [\varepsilon]\times[\sigma] = [\sigma]$ $\quad [\sigma]\times[\sigma] = [1] + [\varepsilon]$

Coulomb gas representation

$$T(z) = +\frac{1}{4} :a(z)a(z): + \alpha_0 \partial a(z)$$

$$a(z)\,a(\omega) \sim \frac{2}{(z-\omega)^2} \qquad \int a(z) = \phi(z)$$

$$c = 1 - 24\alpha_0^2$$

$:e^{\alpha\phi(z)}:$ - conformal weight $\alpha(\alpha - 2\alpha_0)$

$$\alpha_{n,m} = \frac{1-n}{2}\alpha_+ + \frac{1-m}{2}\alpha_-$$

$$h_{n,m} = -\frac{1}{4}\alpha_0^2 + \frac{1}{4}(n\alpha_+ + m\alpha_-)^2$$

$J_\pm(z) = :e^{\alpha_\pm\phi(z)}:$ - conformal dim 1

Fock spaces $\quad F_p = \{ a_0|p\rangle = p|p\rangle, \dots \}$

$:e^{\alpha\phi(z)}:$ - corresponds to highest weight vector α.

Hypothesis:

Conformal blocks:

$$\left\langle V_{\alpha_1}(z_1)\, V_{\alpha_2}(z_2) \ldots V_{\alpha_n}(z_n) \int J_\pm(t_1) \ldots \int J_\pm(t_m) \right\rangle_{-2\alpha_0}$$

total charge.

The integration is not specified yet.

However, this is something very familiar: Homology cycles on the configuration space with coefficients in the line bundle defined by the corresponding multivalued function.