Relation to Quantum Groups. (Lecture IV)

Choose $:e^{\alpha_{n,m}\phi(z)}:$ Such operators satisfy braided VOA. Choose $m=1$

$$:e^{\alpha_{n+1,1}\phi(z)}: = V_n(z)$$

Let us look at the corresponding Fock module F_n.

<u>Proposition</u> $Q = \oint J_-(z)\,dz$ is well defined on F_n and the kernel of this map gives the irreducible Virasoro module $V_{(h_{n+1,1}),c}$; $c = 13 - 6\left(\alpha_+^2 + \frac{1}{\alpha_+^2}\right)$.

<u>Proof.</u> $F_n \xrightarrow{Q_-} F_{\alpha_{n+1,1}+\alpha_-}$

Computing the characters completes the proof.

We want to construct an operator

$$\Phi^{s}_{mn}(z) : V_{h_{m+1,1;c}} \otimes V_{h_{s+1,1;c}} \longrightarrow V_{h_{n+1,1;c}}$$

$$n = m+s-k$$

Construction:

Take $\underbrace{V_{m}(z)\; J_+(x_1) \ldots J_+(x_k)\; V_s(0)}_{\mathcal{F}_h\{z,\{x_k\}\}}$

parameter z

integrate over $x_1, \ldots, x_k$

Here $|z| > |x_1| \ldots > |x_k| > 0$

The multivalued part:

$$\prod_i (z - x_i)^{-\alpha_+^2 m} \prod_{i<j} (x_i - x_j)^{\alpha_+^2} \prod (x_i - 0)^{-s\alpha_+^2}$$

$$(z-0)^{\alpha_+^2 (m \cdot s)}$$

In general.

$\overset{\circ}{z}_1^{\lambda_1} \ldots \overset{\circ}{z}_m^{\lambda_m}$

configuration space

$\mathbb{C}^k / \{z_1, \ldots z_m\}$, diagonal

$$\mathcal{L} = \prod_i (z_i - z_j)^{\alpha_+^2 \lambda_i \lambda_j} \prod_{i,k} (z_i - x_k)^{-\alpha_+^2 \lambda_i}$$

$$\prod_{i<j} (x_i - x_j)^{\alpha_+^2}$$

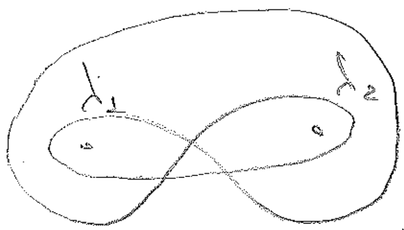
$\leftarrow$ line bundle.

$$H_r^{\Sigma}(\mathbb{C}^r / \{z_1 \dots z_m\}, \Delta; \mathcal{L}) =$$

$$= \operatorname{Sing}_{\lambda_1 + \dots + \lambda_m - 2r} (V_{\lambda_1}^q \otimes \dots \otimes V_{\lambda_m}^q) \qquad q = e^{\pi i \alpha_+^2}$$

α_+^2 - generic.

<u>Example:</u> Given $H_1(\mathbb{C} / \{z_1, z_2\}, \Delta; \mathcal{L}) = \mathbb{C}$



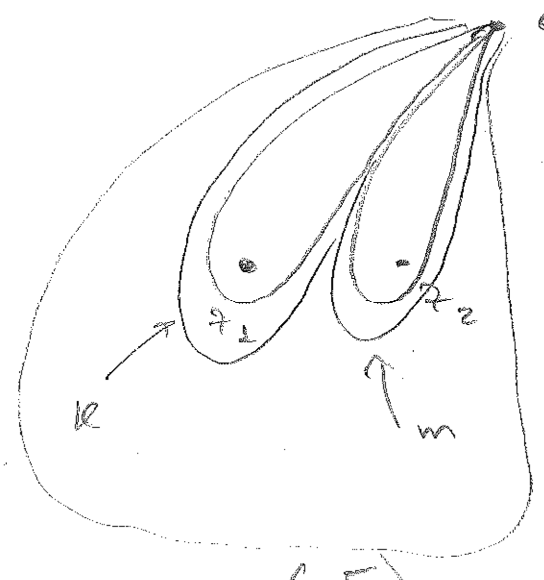
- Pocchammer contour.

$$\Phi_{m\,n}^{s,|s\rangle}(z) = \int_C V_{m'}(z)\, J_+(x_1) \dots J_+(x_k)\, V_{s'}(0)\, dx_1 \dots dx_k$$

$|s\rangle$ ← highest weight

$$C \in H_k^{\Sigma}(\mathbb{C}^k / \{z_1, 0\}, \Delta, \mathcal{L})$$

How does it work?



$\Delta(F)$

$$F^k |\lambda_1\rangle \otimes F^m |\lambda_2\rangle$$

∂ - boundary gives $\Delta(E)$