The vibration of drumhead is model by the Dirichlet eigenvalue problems
\[ \begin{align*}
-\Delta u(x) &= \lambda u(x) & x \in \Omega,
\end{align*} \]
where \( \Omega \) is a smooth bounded domain in \( \mathbb{R}^n \), and it is interesting and fundamental to study the sizes of nodal sets in the vibration. It was conjectured by the Fields medalist Shing-Tung Yau \[1\] that the sizes of interior nodal sets are bounded as
\[ |\{ x \in \Omega : u(x) = 0 \} | \leq C \lambda \]
The interior nodal sets touch the boundary \( \partial \Omega \) to form boundary nodal sets. It was shown by Zhu \[2\] that the sharp upper bound of boundary critical sets is
\[ |\{ x \in \partial \Omega : |\nabla u(x) | = 0 \} | \leq C \lambda \]
Our goal is to verify that the upper bound in (3) and (4) are optimal. We exam the vibration of two-dimensional Chlandi Pattern in the ball
\[ \begin{align*}
-\Delta u(x, y) &= \lambda^2 u(x, y) & (x, y) \in \mathbb{B}_1,
\end{align*} \]
where \( \mathbb{B}_1 = \{(x, y) : x^2 + y^2 \leq 1\} \) and \( \Delta u(x, y) = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \). To solve this equation, we used polar coordinates \( (r, \theta) \) where \( 0 \leq r \leq 1 \) and \( 0 \leq \theta \leq 2\pi \). Utilizing our boundary condition, we began to solve the two-dimensional aspect concerning a disc shaped:
\[ -\Delta u = a_0 + \frac{1}{r^2} + \frac{1}{r} \]
The next step in solving for the nodal lines and curves is to use the separation of variables method as shown below:
\[ u(r, \theta) = R(r)Q(\theta) \]
We simplified the original equation by separating the variables given in the equation. We can now substitute this value into the original equation to give us Bessel's Equation:
\[ r^2 R''(r) + r R'(r) + \left( \lambda^2 r^2 - \ell^2 \right) R(r) = 0 \]
The equation (6) is called Bessel's Equation. The solution of (6) is given as the \( k \)-th Bessel function \( J_k(\ell) \). The solution is as follows:
\[ u(r, \theta) = A_k J_{k}(\kappa r) \cos(\kappa \theta + \theta_0) \quad \text{or} \quad u(r, \theta) = A_k J_{k}(\kappa r) \sin(\kappa \theta + \theta_0) \]
where \( \kappa = \sqrt{\ell^2 - \lambda^2} \) is the root of \( J_k(\ell) = 0 \).

\section*{Vibration of Drumhead: Sizes in the Music}
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\section*{INTRODUCTION}
Chladni Pattern models the nodal patterns in vibrating plates. Ernst Chladni (Father of acoustics, 17561827) introduced the Chladni Pattern using sand and metal plates. By striking the plates, the sand would attract to areas where no vibration occurred, which leads to strange patterns forming.

\section*{Our Research Questions}
To dive deeper into the nodal patterns, we decided to focus on two important research questions which include: How many nodal lines or curves are created and how many nodal lines or curves intersect the boundary? This project focuses on the vibration of circular drumhead and answers the questions discussed above.

\section*{PLAYING GUITAR}
The vibration of guitar string is represented by the ordinary differential equation
\[ \begin{align*}
-\frac{d^2 y}{dx^2} &= \lambda^2 y(x) & x \in (0, 1),
\end{align*} \]
where \( \lambda \) is the frequency. Solving the equation by trying the solutions \( y(x) = e^{\pm \lambda x} \), we arrive at
\[ y(x) = c_1 \cos(\lambda x) + c_2 \sin(\lambda x). \]
The boundary conditions lead to \( y(0) = y(1) = 0 \) for each \( \lambda \) of eigenvalues \( \lambda = k \) and \( k = 1, 2, 3, \ldots \). The specific nodal points \( x = 0, \frac{1}{2}, \frac{1}{3}, \ldots \) are
\[ \begin{align*}
\text{The number of nodal points are} \quad H^2[0, 1][y(x) = 0] = k - 1 \leq CA. \tag{2}
\end{align*} \]

\section*{TWO-DIMENSIONAL CIRCULAR DRUMHEAD AND ITS SOLUTIONS}
Chladni Pattern: Playing Guitar for \( k = 1, 2, 3, 5, 100 \)

\section*{STUDY OF NODAL CURVES}
If \( \sin(kM_0) = 0 \), then \( \theta = \frac{m\pi}{M_0} \) with \( m = 0, 1, \ldots, M_0 - 1 \). Thus the nodal sets of \( u(r, \theta) \) is a collection of and \( s-1 \) concentric circles with radius \( R_0 \), \( R_1, \ldots, R_{s-1} = \).
The length of nodal line from \( M \) diameters is \( 2k \).

\section*{REFERENCES}