Properties of $\ln x$

1. The *domain* is the set of all positive real numbers $x > 0$.
2. The *range* is the set of all real numbers $-\infty < y < \infty$.
3. **Algebraic properties:** If a and b are any positive real numbers, and r is any real number, then
   (a) $\ln 1 = 0$
   (b) $\ln ab = \ln a + \ln b$ (Product rule)
   (c) $\ln \frac{a}{b} = \ln a - \ln b$ (Quotient rule)
   (d) $\ln a^r = r \ln a$ (Power rule)
   (e) $\ln \frac{1}{a} = -\ln a$
4. **Differentiation and Integration:**

$$\frac{d}{dx} \ln x = \frac{1}{x}, \quad \int \frac{1}{x}\, dx = \ln|x| + C, \quad \text{and} \quad \int \ln x\, dx = x \ln x - x + C$$

Properties of e^x

1. The *domain* of the exponential function is the set of all real numbers, $-\infty < x < \infty$.
2. The *range* of the exponential function is the set of all positive real numbers $y > 0$.
3. The exponential function is the *inverse* of the natural logarithm function. This means

$$\begin{aligned} e^{\ln x} &= x \quad \text{for all } x > 0, \text{ and} \\ \ln e^x &= x \quad \text{for all } x \in \mathbb{R}. \end{aligned}$$

4. **Algebraic Properties:**
   (a) $e^0 = 1$
   (b) $e^{x+y} = e^x e^y$
   (c) $e^{x-y} = e^x / e^y$
   (d) $e^{-x} = 1/e^x$
5. **Differentiation and Integration:**

$$\frac{d}{dx} e^x = e^x \quad \text{and} \quad \int e^x\, dx = e^x + C.$$

Trigonometric Functions

1. **Identities**

   (a) *Pythagorean:* $\sin^2\theta + \cos^2\theta = 1, \quad \tan^2\theta + 1 = \sec^2\theta$

   (b) *Parity:* $\sin(-\theta) = -\sin\theta, \quad \cos(-\theta) = \cos\theta$

   (c) *Addition Formulas:*

   i. $\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

   ii. $\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

   (d) *Product Formulas:*

   i. $\sin\theta\sin\phi = \frac{1}{2}(\cos(\theta-\phi) - \cos(\theta+\phi))$

   ii. $\cos\theta\cos\phi = \frac{1}{2}(\cos(\theta-\phi) + \cos(\theta+\phi))$

   iii. $\sin\theta\cos\phi = \frac{1}{2}(\sin(\theta+\phi) + \sin(\theta-\phi))$

   (e) *Amplitude-Phase Shift Formulas:*

   i. $A\cos\theta + B\sin\theta = C\cos(\theta-\phi)$, where $C = \sqrt{A^2+B^2}$ and $\tan\phi = B/A$

   ii. $A\cos\theta + B\sin\theta = C\sin(\theta+\phi)$, where $C = \sqrt{A^2+B^2}$ and $\tan\phi = B/A$

2. **Differentiation and Integration**

$$\frac{d}{dx}\sin x = \cos x \qquad \int \sin x = -\cos x + C$$

$$\frac{d}{dx}\cos x = -\sin x \qquad \int \cos x = \sin x + C$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\sec x = \sec x\tan x$$